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Abstract

The Control design problems with H ∞ bounds are considered.

Given a plant $y = Gu$ (in the classical problem it is assumed linear stationary, but we shall not need stationarity), devise a feedback control $u = Ky$ which shall make the *sensitivity transfer function* $T = (I + GK)^{-1}$ small, in order to be robust, i.e. resistant against model errors in G. The natural meaning of "small", to engineers, refers to the maximum value of (the norm of) $T(i\omega)$ over all frequencies, i.e. the H_∞ norm of T, which has to lie in the Hardy space H_∞ in order for the closed loop system to be stable. Further, for measurement noise insensitivity it would be desirable that the *complementary sensitivity function* $T_c = GK(I + GK)^{-1}$ also be small. However, since $T + T_c = 1$, it is impossible to make both small at a time. The solution to this dilemma is to realize that modelling errors introduce *low frequency* disturbances, while measurement errors tend to be *high frequency.* Hence, the idea of *loop shaping* whereby one attempts to control the magnitude of T, say, at low frequencies, and of T_c at high frequencies. This is usually done by defining weighting functions $W(s)$ and $W_c(s)$, and controlling the norm of the combined transfer function $H = [WT\ W_cT_c]$.

Building on that idea, many more refined problems have been defined, attempting also, for instance, to control the input activity transfer function $K(I+GK)^{-1}$, and so on. Provided that the various weighting functions be taken as rational, sometimes with care to the properness of the various systems involved, all these problems may be cast into the *standard problem* described below.

We offer a slightly different rationale for the current approach, still related to robustness, but closer to the min max approach we adopt here.

Abstract

We consider the following non-zero-sum two-person game. Each of the two players draws a number according to a bivariate distribution on [0,1]. After observing his number each player can then chose to offer or not offer to exchange his number for the other players number. Conditions for an exchange are the following: (1) both players must offer for a trade to occur with certainty; (2) if only one player offers, a trade occurs with probability p. A players payoff is the number he holds after a possible exchange has occurred. Also, a two-stage variant of this game is considered. The optimal strategies of the players are described.

Abstract

The process of pollution propagation in the air due to the wind is considered. There are pollutant sources of different strength located in a finite number of points of a region D. Some two-player games arising in this context are considered. Player 1, by the choice of sources and strength coordinates the pollution emission in an attempt to maximize the quantity of sediments falling over the region X. The aim of a player 2 is opposite. Various statements of this problem are considered. An example is solved.

Abstract

The article contains the solution of the well-known game Black Jack (Russian variant). In the game in the pack of 36 cards each card corresponds to some number of points: jack-2, queen-3, king- 4, ace-11 and to any other card the number marked on it. There are two players taking some cards and calculating the sums of corresponding points. If the sums of corresponding points are less or equal to 21 then the player having the largest sum of the points is the winner. If the sum of only one of the players is more than 21 then his opponent wins. In other cases a draw is declared. The mathematical model of the game and optimal stopping rules for both players are developed in the article.

Abstract

The author proposes a family of strongly time consistent optimality principles (STCOP) for cooperative differential games. Using the regularizing procedure different STCOPs based on core, Shapley value, NMsolution are obtained. Two different approaches are considered. The first is based on the construction of a new characteristic function (and finally constructing an integral optimality principle) and in the second one an imputation distribution procedure which leads to the differential strongly time-consistent optimality principle is defined. Also the author considers the problem of strong time-consistency for the case of the nperson differential games with discount payoffs on the infinite time horizon.

Abstract

The goal of this paper is to show possible approaches of obtaining the upper estimations of the value of differential games with incomplete information. The simple pursuit zero-sum games with prescribed duration T on the plane between the team of pursuers P = P1,...,Pm and the evader E are considered. Both players receive an information about the opponents position with time lag. The upper estimations of the values of two differential games with different payoff functions are obtained and the corresponding pure piecewise constant strategies of the pursuer are described. It is shown how to calculate the minimal number of the pursuers, who can provide the payoff of the player E not exceeding a given number V. Stochastic covers of the reachable sets of the player E are used and the corresponding mixed piecewise constant strategy of the player P is described.

Abstract

Control of Markov chains by a team of agents is a challenging problem both in the team and game situations. In the team case, if the decision makers (DMs) have access only to imperfect-state information, and they exchange this information at every step (i.e., both DMs have common information) or if the underlying information pattern is of the one-step delay type, dynamic programming arguments can be used to obtain the optimal policies. But one has to redefine the Markov chain on an appropriate uncountably infinite state space, corresponding to the sufficient statistics for the problem, before obtaining the optimal solution. If the delay in information exchange is more than one step, then the problem falls in the category of stochastic control problems with nonclassical information patterns, the challenging aspects of which were first pointed out by it H. S. Witsenhausen (1968).

Of course, in the extreme (special) case when the transition probability matrix is block diagonal, with each block controlled by only one decision maker, the problem becomes tractable, since it can (trivially) be decomposed into single decision maker Markov chain control problems, each one of which can be solved independently. If there is a strong interaction between the blocks, however, the problem becomes quite intractable.

We study the intermediate case when the interaction between the blocks of the transition probability matrix is weak, specifically $O(\epsilon)$, where ϵ is a small parameter. By exploiting this weak coupling between the blocks, it is then possible to obtain the optimal solution, provided that the value of ϵ is sufficiently small.

Abstract

This paper discusses a model of tax inspection activity in order to determine the optimal strategy of its organization. We shall consider the following description of interaction among Businessmen, Inspectors and the Center. At the first stage every Businessman is to report to the Center whole sum of his income. In reporting he can either show real sum of the income or show his income below real in order to reduce the tax value. At the second stage every Inspector checks at random some of Businessmen attached to him for a given period. Then the Inspector informs the Center about the established incomes of these Businessmen. This information may be either valid or not. At the third stage the Center checks randomly some of inspectors. Depending on inspection results the Center fines Businessmen exposed of tax evasion as well as Inspectors exposed of bribery or extortion. The information from unchecked Businessmen and Inspectors is treated as valid. We assume that the Center's checking is always correct.

Proceeding from this description the present paper gives a model of tax inspection activity. The paper gives a game theoretic analysis of this interaction in order to determine the optimal values of the above mentioned parameters.

Abstract

A Stackelberg differential game with one leader and n followers is treated. Stackelberg leadership in dynamic games leads to the time inconsistency of optimal policy. To overcome time inconsistency the author proposes to consider t-transferable games. In this type of games optimal value of payoffs of the players are redistributed along the optimal path. A method of regularization (redistribution of payoffs) of the Stackelberg game is proposed. Necessary conditions for time consistency are formulated.

Abstract

An infinite-horizon version of the game investigated by E. G. Enns and the author for the case of a finite possible number of observations is considered. In this article the author obtains a theorem on the existence of an equilibrium point. A specific game related to the best choice problem with full information is investigated also where an explicit form of an equilibrium point is found.

One Approach to the construction of the Time Consistent Optimality Principles in N-Person Differential Games 113–120

D.V. Kuzutin

Abstract

Some optimality principles used in differential games with many players are the attempts of compromise on the set of Pareto-optimal (weak Pareto-optimal) payoffs. An equitable choice of unique solution from these sets may be realized by using known results and methods of the theory of bargaining [1]. Unfortunately, the majority of obtained in this way optimality principles are time inconsistent. Here we offer one approach to the construction of time consistent Kalai-Smorodinsky solution in n-person differential games with integral payoffs.

Single-Sector Stochastic Model of Economics with Perfect Competition 121–128

O.A. Malafeyev and S.A. Nemnyugin

Abstract

In this paper we shall construct two classes of single-sector stochastic models of economics with perfect competition. The first one is created by inclusion of the stochastic evolution of exogenous variables into the well known deterministic model. The perfect competition requires maximization of the income at any moment. The resulting linear stochastic differential equation may be solved by standard methods. The second approach is formulated in terms of the stochastic control. It requires optimization of the stochastic income functional with the supposition that exogenous variables follows stochastic differential equation. This problem may be solved by dynamic programming method. Scheme of investigation and possible generalizations are discussed.

Game-Theoretic Model of Preference 129–137

V.V. Mazalov

Abstract

The following game P may be proposed as a simple model of preference. Suppose that two players, I and II, receive cards x and y, respectively. The card z is dealt to the pile. The random variables x, y, z are independent and uniformly distributed at the interval [0,1]. In the first step, Player I has two possibilities. First, he may take the card from the pile, keep the card with the greater value, and discard the card with the lower value to the discard pile. Then the players open their cards. If the value of Player Is card exceeds the value of Player IIs card, Player I then receives quantity A from Player II; if the value of Player Is card is less than Player IIs card, then Player I pays quantity B to Player II; if the values of the cards are equal, there is no payment. The second possibility in the first step is that Player I can say pass; the game then goes over to the second player.

In the second step, Player II also has two possibilities. Player II may take the card from the pile, keep the card with the greater value and discard the card with the lower value. After Player II does so, the players open their cards. If the value of Player IIs card exceeds the value of Player Is card, Player II receives quantity A from Player I; if the value of Player IIs card is less than the value of Players I card, Player II pays Player I of quantity B; if the values are equal, there is no payment. The second possibility in the second step is that Player II may say pass. In this case the pile remains reserved, the players open their cards and now the situation is reversed. At this point the player wins who has the lower value. In other words, if x¡y, Player I receives quantity C from Player II; if x¿y, Player I pays quantity C to Player II; and, finally, if x=y, there is a draw.

We determine the optimal strategies in this game.

Abstract

We consider game-theoretic model of technology cartels. The model is formalized as a discrete-time infinite horizon dynamic game with non-linear demand and non-linear learning functions. We compute collusive as well as noncooperative solutions. The latter are defined as feedback Nash (subgame perfect) equilibria. The solutions presented in the paper correspond to a wide range of parameter values and, therefore, provide a good insight into the behavior of firms sharing technical knowledge and into the impact that the creation of a TSC can have on industrial concentration and social welfare.

Abstract

In the paper the cooperative differential games with prescribed duration are considered. It is well known that solution concepts taken from classical cooperative static game theory are time-inconsistent and the multi-value solutions are also strongly time inconsistent. The author proposes two methods for regularization of this solutions to insure strongly time consistency. One is based on integration of classical solutions along the so called cooperative trajectory the other uses as solution locally optimal behaviors. The proposed regularization requires in both cases different transformation procedures for corresponding classical characteristic functions.

Abstract

The two-person simple exchange game (**SEG**) is described as follows: Each of two players (I and II) draws a number (x and y, respectively) according to a uniform distribution on $[0, 1]$. After observing his number each player can then choose either to keep (K) his number or to exchange (E) it for the other player's number. If only one player chooses E, an exchange of the numbers occurs with probability p. A player's payoff is the number he holds after the players have made their choices and a possible exchange has occurred. In the present paper we shall investigate **SEG**s in which one or both of the players have various patterns of information-private or public-about his own and opponent's hands. We shall also discuss three-person **SEG**, as a natural extension of the two-person game-model.

Abstract

A pursuit game with one pursuer P and one evader E is considered. E is moving from the origin of the coordinate system O on the plane, checked by straight lines (the lattice with the side of unit length), motion taking place only along the lattice. Two properties of controls in pursuit games are considered in the paper. On the first property: its essence is that in the wide class of pursuit games optimal strategy exists only for the evader. In this case the game in which the phase space and dynamics of the player P is analogous with player E is considered. We prove that in this game for any ξ ¿ 0 there exists the situation of ξ-equilibrium and optimal behavior strategy of player E. On the second property: the phase space of the player E is the same but the dynamic is different. Necessary and sufficient conditions for optimal strategy of the player E are received.

Abstract

In this paper we study a game-theoretical approach to the problem of protecting the environment against pollution by harmful industrial wastes or other sources of pollution. This approach is illustrated by an example of pollution of a region by several enterprises; here is assumed that they are guided by different interests in the implementation of arrangements for protecting the natural environment against pollution.

A static hierarchical model is considered in which the payoffs of the leader (authority) and the players-followers (enterprises) depend on the total discharge and individual (for the enterprises) discharges of pulluting substances, the question of the existence of a Stackelberg sollution in pure strategies is investigated, and method for finding solutions is proposed.

A dynamical model is constructed on the basis of a partial differential equation that describes the spreading of a polluting substance. The payoff functionals of the players depend on the level of pollution and intensity of discharge of earch of the sources. For dynamical model problem of time consistency of the cooperative solution is investigated and method of regularization of this solution is proposed.

Abstract

The paper deals with a two-player bargaining situation in which a limited quantity of goods (with given minimal price) is to be distributed and the sum of players' demands is higher than the disposable quantity of goods. The strategies of players, described by the offered prices, lead to the result satisfying first the demand of the higher bid. The existence of the Nash equilibrium and Pareto optimum in a corresponding bimatrix game modeling the described bargaining is proved. An analytic condition is derived under which the special type of Nash equilibrium exists and the Pareto optimum is unique in pure strategies.